An Investigation into the Use of a Single Zone Map Projection for Illinois

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Abstract: The nature of surveying lends itself to the use of coordinate systems for calculating the positions of points. Large, statewide projects benefit from a single uniform coordinate system that provides one datum for the calculations. Unfortunately, using coordinates from a rectangular system to depict positions from the curved earth's surface leads to scale distortions. Typically, the established state plane coordinate systems have been derived to keep the relative scale error at 1:10,000 or better. This was chosen because the desired and achievable relative error in surveys was thought to be 1:10,000.

Currently, Illinois uses two zones for the state plane coordinate system. A single zone for Illinois would eliminate the calculations required when working from one zone to another. Using appropriate data, a system can be created that would be useful to mapping professionals and land surveyors.

Mapping Systems

A map is a two-dimensional representation of a three-dimensional object. Because of the need to depict a spherical object on a flat surface, there will be distortion of some aspect of the map. The map is a projection of the points on the surface of the earth to a simple mathematical three-dimensional surface, such as a plane, a cylinder or a cone. This projection onto a developable surface can then be viewed in two dimensions. A rectangular coordinate grid system can be superimposed over this surface, creating a map projection. A point on the mapping surface will represent each point that exists on the actual surface.

The calculations that surveyors typically perform are less complicated if they are made using the mapping projection's coordinates instead of the geodetic coordinates. Geodetic coordinate calculations involve complex spherical geometry equations; plane coordinate system calculations can use plane trigonometry. The calculations must be made after the allowance is made for the map distortions and the height above (or below) the ellipsoid the measurements were made.

The projection will determine the relationship between the mapped points. A conformal projection will retain the shape of the true surface; thus, the actual relative angles between points will be the same on the map. An equal area projection will have shape distortions, but will accurately depict the area of the mapped region. An azimuthal projection will have the true angles from one point to another depicted.

There are many different projections, created for any number of uses. An excellent Internet website for visualizing the differences between the various projections can be found at: http://www.btinternet.com/~se16/js/mapproj.htm . One is able to see a map of the globe depicted in various projections.

Transverse Mercator Projection

The orientation and the size of the area to be mapped determine which projection method is the most desirable. A region that is longer in the north-south axis that it is wide in the east-west direction would have less distortion if the area were mapped using the Transverse Mercator Projection. Johann Heinrich Lambert created this map projection method; Carl Friedrich Gauss and L. Kruger made further improvements. Thus it is also termed the Gauss-Kruger Projection. This projection utilizes an idealized cylinder for the one-to-one correspondence of points from the ellipsoid to the map surface.

The relationship between the true surface and the map projection, or scale factor, changes as the distance is increased east or west of the central meridian. For this projection, the shape of a region is preserved in the one-to-one projection of points from the surface to the map; however, the area is not shown accurately on the mapped surface.

Depending on the orientation and size of the state, different projections have been used for mapping. Thus, different bases were used for state plane coordinates. Illinois is mapped using the Transverse Mercator Projection system. Officially, the State is currently mapped in two state plane coordinate zones. The zones are defined and established by law, 765 ILCS 225/1, which is included in the attached Appendix A.

The Transverse Mercator projection is defined as a secant case for a cylindrical projection. The imaginary cylinder has its axis of rotation placed in the plane of the equator of the earth. This results in two circles intersecting the globe, at an equal distance from the central meridian of the projection. The direction of the grid North is determined by the central meridian, which is chosen to lie in the center of the area to be mapped. The map projection that is generated will have curves for all of the parallels of latitude and for all of the meridians except for the central meridian. The northing and easting grid system for the mapping coordinates is defined by axes offset from and parallel with the central meridian. The resultant distortions of the actual distances, areas, directions and scale increase as the distance to the central meridian increases. The Universal Transverse Mercator Projection is used to depict a large portion of the earth, broken down into six-degree sections, from 80 degrees South latitude to 84 degrees North latitude.

Direct and Inverse Calculations

Distances and angles that are measured upon the surface of the earth must be adjusted in order to correlate them to coordinates within a mapping system. The measured distances must be converted to grid distances by applying the appropriate scale and elevation factors. For long lines and very precise surveys, the correction for the scale factor should be evaluated at both end point and the midpoint of the line being surveyed. The angular relationships must be adjusted by the analysis of the convergence angle and the arc-to-chord correction (for long lines and precise surveys). This allows for the conversion from measured geodetic azimuths (or directions) to grid azimuths.

The calculations that are called the *direct* problem take geodetic coordinates (latitude and longitude) and determine the mapping system values (state plane coordinates); and the *inverse* problem, which utilizes the mapping system numbers (state plane coordinates) and transforms them into geodetic coordinates (latitude and longitude). In the Transverse Mercator projection system, the required calculations are complicated to perform by use of calculator. The direct and inverse equations are shown in Appendix C. There are several programs available that will allow for rapid and accurate calculations of both the direct and inverse problems. A convenient Internet website can be found at The National Geodetic Survey at http://www.ngs.noaa.gov/PC_PROD/pc_prod.shtml - SPCS83, or http://www.ngs.noaa.gov/TOOLS/index.shtml - SPCS83. A sample of a spreadsheet used for calculations is included as Appendix D.

State Plane Coordinates

The creation of the first state plane coordinate system was completed by the U.S. Coast and Geodetic Survey (now the National Geodetic Survey) in 1933, for North Carolina. This system, which was performed for the other states, was measured in relation to the NAD27 reference datum. This coordinate system is based on the Clarke 1866 ellipsoid, and is known as the *State Plane Coordinate System of 1927* (SPCS27). The mathematical model had the following parameters:

Ellipsoid	Semimajor axis *a*	Semiminor axis *b*	Flattening *f*	Eccentricity *e*
Clarke 1866	6,378,206.4m	6,356,583.8m	1/294.978698214	0.082271854

The Illinois law, included in Appendix A, specifies the origin coordinates for NAD27 as x = 500,000 feet and y = 0 feet.

Currently, most calculations are made using the NAD83 parameters. Although the projections for most states are approximately the same, the revised parameters will produce different coordinates. The later parameters are as follows:

Ellipsoid	Semimajor axis *a*	Semiminor axis *b*	Flattening *f*	Eccentricity *e*
GRS80	6,378,137.0m	6,356,752.3m	1/298.257222101	0.081819191

Using these parameters and the revised origin coordinates will create coordinates in the *State Plane Coordinate System of 1983* (SPCS83). For Illinois, the offsets are x = 700,000 meters (2,296,583.333 feet) and y = 0 meters for NAD 83 for the West zone and x = 300,000 meters (984,250.000 feet) and y = 0 meters. The “y” or Northing values are sufficiently different to enable one to recognize which system is being used.

Creating a New System

When creating a new system it is necessary to utilize the projection type that best fits the area to be mapped. For the State of Illinois, this is the Transverse Mercator Projection. We need then to establish the allowable scale factor between grid and geodetic measurements. The new system will have to cover an area large enough to allow overlap into adjacent zones. This overlap is necessary for the calculations of surveys that cross into the adjoining states. The choice of the appropriate point of origin for the grid will allow for the best possible map coverage of the selected area. The false northing and easting values must be selected so there will be no confusion as to which projection a coordinate pair belongs. After all of these decisions are made, the tables for the comparisons between geodetic and grid positions and the corrections for azimuths and distances can be prepared.

The central meridian for the East zone, established at 88°−20′W, has a defined error of 1:40,000 too small, or a scale factor of 0.999975. The zone extends east and west 28 miles to meridians that have exact scale (scale factor equals 1.000000000). The central meridian for the West zone, established at 90°−10′W, is set at a scale factor of 0.9999411765, or 1:17,000 too small. The zone width of 42.9 miles east or west of this central meridian provides meridians with exact scale, at 1.000000000. These two zone swaths are the required two-thirds coverage, with overlapping occurring beyond the exact scale meridians at scale factors greater than one.

For the West zone, the distance from the central meridian to either meridian of exact scale is approximately 226,512 feet. For the East zone, it is approximately 147,840 feet. Selecting a new central meridian at $89^0 - 22' - 30''$ would approximately bisect the state. Establishing the two-thirds zone width at 453,000 feet produces a scale error of approximately 1:8,500 or a scale factor of 0.9998823529. The full-zone width (approximately 129 miles) should provide sufficient overlap coverage into both adjacent states' zones. This would extend from $87^0 - 03' - 30''\ W$ to $91^0 - 41' - 30''W$, and would create scale factors of approximately 1.000082 at the limits of the zone. This implies a precision of 8.2/100,000 = 1:12,000. At these limits, the convergence angle would be $0^0 - 45' - 45''$, which is acceptable. This implies that the new scale factor at the central meridian could be increased from 1:8500 to possibly 1:10,000 = 0.9999.

A new system will need to have a revised system of false north and east values for the state plane coordinates to be recognizable and distinct from the other systems in use. The values are selected to be so large that there will never be a negative value. The SPCS83 values were selected using a false easting of 300,000 m for the East zone and 700,000 m for the West zone. The y (North) coordinate value for the origin is zero for all of the cases; the origin is south of the south-most part of Illinois. A system could be created that has a large false Northing value, and some value for x that is different from those used previously. For the purpose of this paper, we will assign the value for y at the origin to be 1,000,000 m, and assign the false x value of 1,000,000 m.

The solution that has been selected by this paper has the following parameters:

Latitude of Origin	36°-40'N
Central Meridian	$89^0 - 22' - 30''$ W
Scale factor at Central Meridian	0.9999
Origin false easting	1,000,000 m
Origin false northing	1,000,000 m
Ellipsoid	GRS80
Flattening *f*	1/298.257222101
Semi-axis: major/minor	6,378,137.0m/6,356,752.3m
Scale factor at 2/3 zone width	0.99995882

A "Microcam"-generated map of Illinois is being included in Appendix B as an attachment. Dr. Scott Loomer of the U.S. Military Academy created this shareware program; the program can be accessed at:

http://hum.amu.edu.pl/~zbzw/glob/microcam.htm or http://www.realearth.com/0600-Tech_Notes_Pages/024-Free_MicroCAM.html

Conclusions: A perfect map projection would faithfully reproduce the true area of a surface, the exact shape of an area and the actual distances between all of the points on the surface. The scale would be the same everywhere on the map as it is on the ground, and the shortest distance between two points would actually be a straight line and not appear as an arc. As this is not physically possible, choices must be made to determine which map projection fills the needs for the selected area. The selection will induce distortions in the map, which are adjusted by the use of certain reduction formulas. A single zone map system for Illinois can be selected that is useful to many, including land surveyors.

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Appendix A
The Illinois Coordinate System Act

(765 ILCS 225/1)

Sec. 1. This Act shall be known and may be cited as the “Illinois Coordinate System Act”.

(Source: P.A. 83-742.)

(765 ILCS 225/2)

Sec. 2. The system of plane coordinates which has been established by the United States Department of Commerce, National Oceanic and Atmospheric Administration, National Ocean Service, National Geodetic Survey for defining and stating the positions or locations of points on the surface of the earth within the State of Illinois is hereinafter to be known and designated as the “Illinois Coordinate System”.

(Source: P.A. 92-311, eff. 8-9-01.)

(765 ILCS 225/3)

Sec. 3. For the purpose of the use of the Illinois Coordinate System, the State is divided into an “East Zone” and a “West Zone”. The area now included in the following counties constitutes the “East Zone”: Boone, Champaign, Clark, Clay, Coles, Cook, Crawford, Cumberland, DeKalb, DeWitt, Douglas, DuPage, Edgar, Edwards, Effingham, Fayette, Ford, Franklin, Gallatin, Grundy, Hamilton, Hardin, Iroquois, Jasper, Jefferson, Johnson, Kane, Kankakee, Kendall, Lake, LaSalle, Lawrence, Livingston, McHenry, McLean, Macon, Marion, Massac, Moultrie, Piatt, Pope, Richland, Saline, Shelby, Vermilion, Wabash, Wayne, White, Will and Williamson.

The area now included in the following counties constitutes the “West Zone”: Adams, Alexander, Bond, Brown, Bureau, Calhoun, Carroll, Cass, Christian, Clinton, Fulton, Greene, Hancock, Henderson, Henry, Jackson, Jersey, JoDaviess, Knox, Lee, Logan, McDonough, Macoupin, Madison, Marshall, Mason, Menard, Mercer, Monroe, Montgomery, Morgan, Ogle, Peoria, Perry, Pike, Pulaski, Putnam, Randolph, Rock Island, St. Clair, Sangamon, Schuyler, Scott, Stark, Stephenson, Tazewell, Union, Warren, Washington, Whiteside, Winnebago and Woodford.

(Source: P.A. 83-742.)

(765 ILCS 225/4)

Sec. 4. As established for use in the East Zone, the Illinois Coordinate System is named, the “Illinois Coordinate System, East Zone”.

As established for use in the West Zone, the Illinois Coordinate System is named, the “Illinois Coordinate System, West Zone”.

(Source: P.A. 83-742.)

(765 ILCS 225/5)

Sec. 5. The plane coordinates of a point on the earth’s surface, used in expressing the position or location of that point in the appropriate zone of this system, consists of 2 distances, expressed in units of U.S. survey feet and decimals of a foot. One of these distances, known as the “x-coordinate”, gives the position in an east-and-west direction; the other,

known as the “y-coordinate”, gives the position in a north-and-south direction. These coordinates depend upon and conform to the coordinates, on the Illinois Coordinate System, of the monumented survey stations of the United States National Geodetic Survey within the State of Illinois, as those coordinates have been
determined by that survey.

(Source: P.A. 92-311, eff. 8-9-01.)

(765 ILCS 225/6)

Sec. 6. When any project extends from one into the other of the above coordinate zones, the positions of all points on its boundaries may be referred to either of the two zones, the zone which is used being specifically named for the project.

(Source: P.A. 83-742.)

(765 ILCS 225/7)

Sec. 7. For purposes of more precisely defining the Illinois Coordinate System the following definitions by the United States National Geodetic Survey are adopted: The Illinois Coordinate System, East Zone, is based on the transverse Mercator projection of the North American Datum of 1983 (NAD 83) or the Clarke spheroid of 1866 (North American Datum of 1927) (NAD 27), having a central meridian of eighty-eight degrees and twenty minutes West (88°–20’W.) of Greenwich on which meridian the scale is set at one part in 40,000 too small. The origin of coordinates is at the intersection of the meridian eighty-eight degrees and twenty minutes West (88°–20’W.) of Greenwich and thirty-six degrees and forty minutes North (36°–40’N.) latitude. The origin is given the coordinates x = 300,000 meters (984,250.000 feet) and y = 0 meters for NAD 83 and x = 500,000 feet and y = 0 feet for the NAD 27.

The Illinois Coordinate System, West Zone, is based on the transverse Mercator projection of the North American Datum of 1983 (NAD 83) or the Clarke spheroid of 1866 North American Datum of 1927 (NAD 27), having a central meridian of ninety degrees and ten minutes West (90°–10’W) of Greenwich, on which meridian the scale is set at one part in 17,000 too small. The origin of coordinates is at the intersection of the meridian ninety degrees and ten minutes West (90°–10’W.) of Greenwich and thirty-six degrees and forty minutes North (36°–40’N.) latitude. The origin is given the coordinates x = 700,000 meters (2,296,583.333 feet) and y = 0 meters for NAD 83 and x = 500,000 feet and y = 0 feet for the NAD 27.

The position of the Illinois Coordinate System is as marked on the ground by monumented survey stations established in conformity with standards adopted by the United States National Geodetic Survey for second and higher order work, whose geodetic positions have been rigidly adjusted on the North American Datum (NAD 1927 or NAD 1983, or both), and whose coordinates have been computed on the system herein defined. Any such stations may be used for establishing a survey connection with the Illinois Coordinate System.

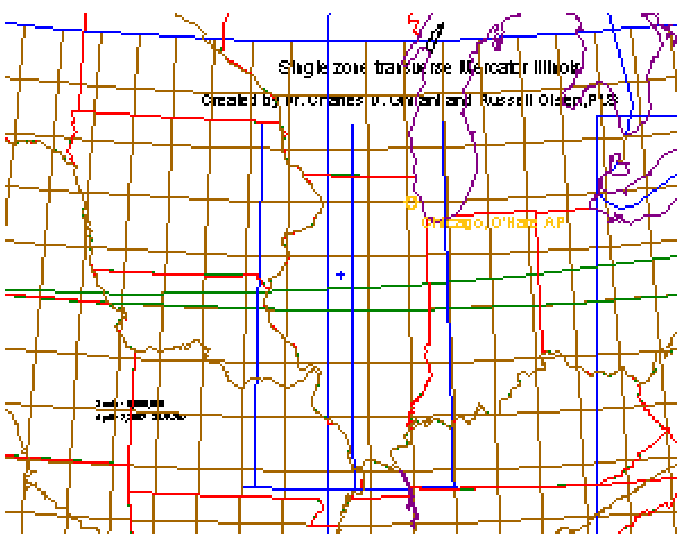
(Source: P.A. 92-311, eff. 8-9-01.)

(765 ILCS 225/8)

Sec. 8. The use of the term "Illinois Coordinate System" on any map, report, survey, or other document is limited to coordinates based on the Illinois Coordinate System as defined in this Act. Any land survey referenced to the Illinois Coordinate System must indicate the Zone and delineate on the plat of survey all geodetic stations, azimuths, angles and distances used for establishing the survey connection.

(Source: P.A. 83-742.)

Appendix B
"Microcam"-generated map of Illinois



Appendix C
Direct and Inverse formula for the Transverse Mercator Projection
Direct Problem

$$\Delta\lambda'' = (\lambda_b - \lambda)(3{,}600'')/\circ$$

$$p = 10^{-4}\Delta\lambda''$$

$$N_p = (I) + (II)p^2 + (III)p^4 + N_b$$

$$E_p = (IV)p + (V)p^3 + (VI)p^5 + E_0$$

The values for I, II, III, IV, V, and VI are interpolated from tables for the projection.

Inverse Problem

$$E' = E - E_o$$

$$N' = N - N_b$$

$$q = E' \times 10^{-6}$$

$$\phi_f = \phi_{table} + \Delta\phi''$$

$$\Delta\phi'' = -(VII)q^2 + (VIII)q^4$$

$$\Delta\lambda'' = -(IX)q + (X)q^3 + (XI)q^5$$

$$\phi = \phi_f + \Delta\phi''$$

$$\lambda = \lambda_b + \Delta\lambda''$$

The values for VII, VIII, IX, X, and XI are interpolated from tables for the projection.

Appendix D
Spreadsheet example of Calculations

Ellipsoid parameters

Name =	GRS 80
	6,378,137.00
	298.2572221009
	0.006694380
	0.081819191
	0.006739497

Steps:

- Complete the cells below using the "Lambert Conformal Conic" and "SPCS 83 using TM" worksheets for assistance.
- Copy values from the other sheets and use the "Paste Special – paste values" feature in the Edit menu.

Endpoint coordinates

	From	To		
Name	Red	Blue	**Mean**	**Δ-Coord**
x (m)	126,703.0681	129,253.0681	127,978.068	2,550.000
y (m)	22,902.2324	27,318.9620	25,110.597	4,416.730

Distance Reduction

			Optional km		
k	0.999906680838	0.999905298308			
Elevation	375.592	354.785	365.189	-20.807	
Geoid height	-31.760	-31.743	-31.752		
Horizontal distance	5,100.747	m			
latitude, ϕ, of From station	39	2	21.63632	0.682836633	
Grid azimuth of line	30.0000002	30	0	0.00	0.5236
Rm	6,360,853.786	m			
Rn	6,386,654.201	m			
Rα	6,367,284.328	m			
Elevation Factor	0.999947635519				
Ellipsoid distance	5,100.480	m			
Scale factor	0.999905989573	grid factor	0.999853630015		
Grid distance	5,100.000	m			

Arc-to-Chord correction

E0 =	600,000
k0 =	0.99995684

Lambert Conformal Conic

sin ϕ0 =	0.661539733812				
ϕ1 =	0.721374445	radians	ϕ2 =	0.72140051	radians
λ1 =	-1.326617718	radians	λ2 =	-1.325577012	radians
ϕ3 =	0.721383133	radians			
δ"	-0.12	seconds			

Transverse Mercator

ϕf =	0.681370499	radians

ηf^2 =	0.005234636	
E3 =	-1,417,340.796	
F =	2.46469E-14	
δ"	5.29	seconds